RESEARCH PAPER

# Evaluating Factors that Influence Young Children's Attitudes Towards Mathematics: The Use of Mathematical Manipulatives

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The use of manipulatives to develop conceptual understanding appears to be a prevalent practice in many mathematical learning experiences, particularly in the early years of schooling. This study evaluates the impact of mathematical manipulatives on young children's attitudes towards mathematics (YCATM). The modified three-dimensional model of attitude (MTMA) and Bruner's experiential stages were used to investigate how manipulatives influence YCATM. The findings suggest that young children enjoyed using manipulatives, contributing to their Vision of Mathematics and Perceived Competence. However, the transition between enactive, iconic, and symbolic experiences can contribute to the formation of negative attitudes.

Children's attitudes towards mathematics are strongly related to their receptiveness to learning and understanding mathematics, their achievement, the value of the subject, self-confidence, and enjoyment (Stiles et al., 2008). Underlining the necessity for mathematics and how children develop mathematical understanding is the need to understand the factors that influence attitudes towards mathematics. Attitude is a multi-dimensional construct with affective, cognitive, and behavioural dimensions (Walker et al., 2020). Investigating attitudes towards mathematics as a multi-dimensional construct provides an erudite view between attitudes and mathematics achievement (Walker, 2020). Connected to this is the phenomenon of negative attitudes and mathematics anxiety. Before these terms, Gough (1954) used "Mathemaphobia" and believed that many failures in mathematics are attributed to a phobia of mathematics. Gough (1954) claims "victims" suffering the phobia avoid studying mathematics and proposes addressing the issue by identifying causes and influences. For this reason, it is crucial that attitudes towards mathematics and the factors be understood so that positive attitudes can be fostered and nurtured. While there is a wealth of knowledge about older students' attitudes towards mathematics, in the case of young children's attitudes towards mathematics (YCATM), there is a dearth of research (Ingram et al., 2020).

The dearth of research extends to investigating how the use of manipulatives by children influences their attitudes towards mathematics. Manipulatives are an established mathematics education resource that can be a "positive tool to improve student learning" (Liggett, 2017, p. 90) and a tool to develop conceptual understanding in mathematics (Quane & Brown, 2022). Further, manipulatives are an established form of mathematical representation (Moyer, 2001). Goldin and Shteingold (2001) remarked: "that a mathematical representation cannot be understood in isolation" (p. 1). Rather, a representation of mathematics is part of a more comprehensive system of mathematical conventions and meaning. Mathematics representations can be a process and a product and are broadly classified as external or internal representations (Goldin & Shteingold, 2001), with manipulatives being an example of a product and an external representation. However, research indicates that manipulatives are more than physical, external representations. Bruner (1996) suggests that our world can be represented and translated into experience in three stages: enactive (action), iconic (perceptual organisation), and symbolic (words and symbols). Research has focused on how teachers effectively use manipulatives to facilitate mathematical learning (Quane & Brown, 2022; West, 2018) and the challenges of using manipulatives (Moch, 2002) and warn that manipulatives do not necessarily lead to success and can even be detrimental to learning (McNeil & Jarvin, 2007). Few studies, however, have explored the use of manipulatives from a child's perspective

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and how they influence attitudes towards mathematics. Given the dearth of research, this study investigated the range and nature of YCATM and, in doing so, identified a range of factors that were found to influence attitudes. Numerous factors were identified, including, but not limited to, the use of technology, game-based pedagogies, tests and assessments and manipulatives. The focus of this paper is on the use of manipulatives and how they influence attitudes towards mathematics. The guiding research question is:

> *How do manipulatives used during mathematical learning experiences influence young children's attitudes towards mathematics?*

## Theoretical Framework

To explore the confluence of YCATM and the use of manipulatives, the Modified Three-dimensional Model of Attitude (MTMA) was used to define the construct of attitudes towards mathematics (Quane et al., 2021). Bruner's (1966) experiential stages of learning were used to analyse how children used manipulatives and their attitudinal response to using manipulatives. To further categorise and develop a more nuanced understanding, the mapping mathematical materials framework by Larkin (2016) was applied to describe how children used manipulatives (Table 1).

Table 1
*The Modified Three-dimensional Model of Attitude (MTMA) with Reference to Bruner's Experiential Stages of Learning*

| MTMA Dimension | Bruner's Experiential Stages of Learning |
|---|---|
| ED: Emotional Tendency | Children's initial emotional response and reaction during enactive learning experiences using manipulatives and emotions towards iconic and symbolic representations of manipulatives. |
| ED: Overall Sentiment | Children's general reactions and emotional beliefs regarding mathematics, including non-verbal cues (posture, gestures and body language) and verbal cues to the use of manipulatives (enactive, iconic and symbolic representations of experience) |
| VM: Topics, Tasks and Processes | Types of mathematical learning experiences and processes identified by children; the number of mathematical topics and how children communicate their mathematical understanding and learning. For example, children's use of manipulatives during mathematical learning experiences (enactive); children's drawings of manipulatives to represent mathematical concepts, ideas, and thinking (iconic); children using or discussing the use of manipulatives to then represent mathematics in written form using words and symbols (symbolic). |
| VM: Value and Appreciation | How and what children view as important and acknowledge as worthwhile about mathematics. For example, what worth or importance do children place on using manipulatives in mathematics as a direct sensory experience (enactive), using pictorial representations of manipulatives (iconic), using manipulatives to aid in the representation of mathematics abstractly (symbolic). |
| PC: Mathematical Mindset | Children's mathematical mindset and perceptions of themselves related to their ability to do mathematics. Children's mathematical mindset when using manipulatives, drawing, or making iconic representations of their use of manipulatives or recording their use in symbolic form. |
| PC: Self-concept | Children's beliefs in their mathematical ability and their expectancy for success when using and representing manipulatives. |

The MTMA moves beyond the dichotomies of "liking" versus "disliking" (Capps & Cox, 1969) and "positive" versus "negative" (Lipnevich et al., 2013) to capture the complexity of attitudes in three broad dimensions. These three broad and interconnected dimensions were

conceptualised by Di Martino and Zan (2010) and encompassed the Emotional Disposition (ED), Vision of Mathematics (VM) and Perceived Competence (PC) dimensions. The MTMA provides six explicit sub-dimensions that can be used to classify and describe YCATM, placing a premium on the *developmental* aspects of children (Quane et al., 2021). Further, moving the definition of attitude away from a dual classification system to include a more extensive and nuanced description of attitude affords the opportunity to also identify factors that influence their attitudes. Each original dimension of the original TMA was modified to include two sub-dimensions, as shown in Table 1. The six sub-dimensions of MTMA were used to identify how the use of manipulatives influence YCATM. A manipulative is "an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (Swan & Marshall, 2010, p. 14). This paper reports on the use of physical manipulatives.

## Method

A mixed-methods methodology was used to investigate the influence of manipulative use on YCATM. Qualitative research techniques involved using children's drawings, written descriptions, interviews, and observations. These methods include the use of children's drawing, and the affordances and limitations (see Quane et al., 2021) have been previously presented. The use of child-centric research techniques afforded children to express what is important to them regarding their mathematical learning. Quantitative analyses were conducted first to identify the range of attitudes, followed by qualitative analyses to describe the nature of attitudes. The frequency and distribution of scores provide the *range* of children's attitudes. The *nature* of children's attitudes is a narrative that has been developed based on acts, actions, artefacts, actors, and significant events that children discussed during the interview when talking about their drawing and during the mathematical learning experiences observations. Three phases of data collection and analyses were enacted: an exploratory study ($n$ = 25); main study ($n$ = 81) and overall data analysis ($N$ = 106) involving systematic, numerical, thematic, and comparative analyses. Children's drawings were coded using and inductive, deductive, and anticipatory responses (Quane et al., 2021). The 106 Year 2 and 3 children attended three South Australian schools and were from ten classes. The data generated has been re-analysed using open coding and TMA and Bruner's experiential stages to explicate how manipulatives influence YCATM. After the generated data were coded, indicators were developed aligning to the framework outlined in Table 1.

This research examined YCATM and the factors that influence attitudes in both lesson and non-lesson contexts, bridging the gap in researching attitudes towards mathematics. Children's drawings, written descriptions, and interview responses ($N$ =106) were collected in a non-lesson context. That is, data collection occurred with children in a non-classroom environment. All children were assigned an alpha-numerical code to ensure anonymity. The letter indicates which school the child attended, and the number indicates the order in which the drawing and interview was conducted. Observations of mathematical learning experiences ($n$ = 27) were conducted after the non-lesson data collection. In the lesson context, three children from each class were purposefully selected based on their attitude classification from the non-lesson context and observed during mathematical learning experiences. Children's attitudes were classified for both the lesson and non-lesson contexts. In reporting the findings, the context where the data were generated is indicated providing transparency regarding the source of data. The observations focused on overt behaviours that were observable by an onlooker. It is not that concealed or non-observed actions, reactions or behaviours are considered inconsequential. Rather, they are hard to detect in a classroom situation. Unknown circumstances may have impacted what the children were experiencing on the day they completed their drawing and the days of the observed mathematical learning experiences.

## Findings

Mathematics representations emerged from the non-lesson context as a theme of interest, with 41 (39%) children discussing the use of manipulatives. During the non-lesson context, children discussed using manipulatives such as unifix cubes, dice and counters to represent and solve mathematical questions and problems. Figure 1 includes selected children's drawings that depict the use of manipulatives. Additionally, children went beyond identifying different manipulatives to describe how they used manipulatives in the context of mathematical topics, their emotions towards using particular manipulatives, and their perceived competence in using the manipulative. During mathematical learning experiences, children's use of manipulatives was observed to see how their attitudes towards manipulatives were enacted. A more extensive range of manipulatives was noted in the lesson context, including unifix cubes, dice, counters, geoboards, Polydrons, measuring instruments, attribute blocks, paddle pop sticks, dice, clocks, number charts, and tens frames, were associated with doing mathematics.

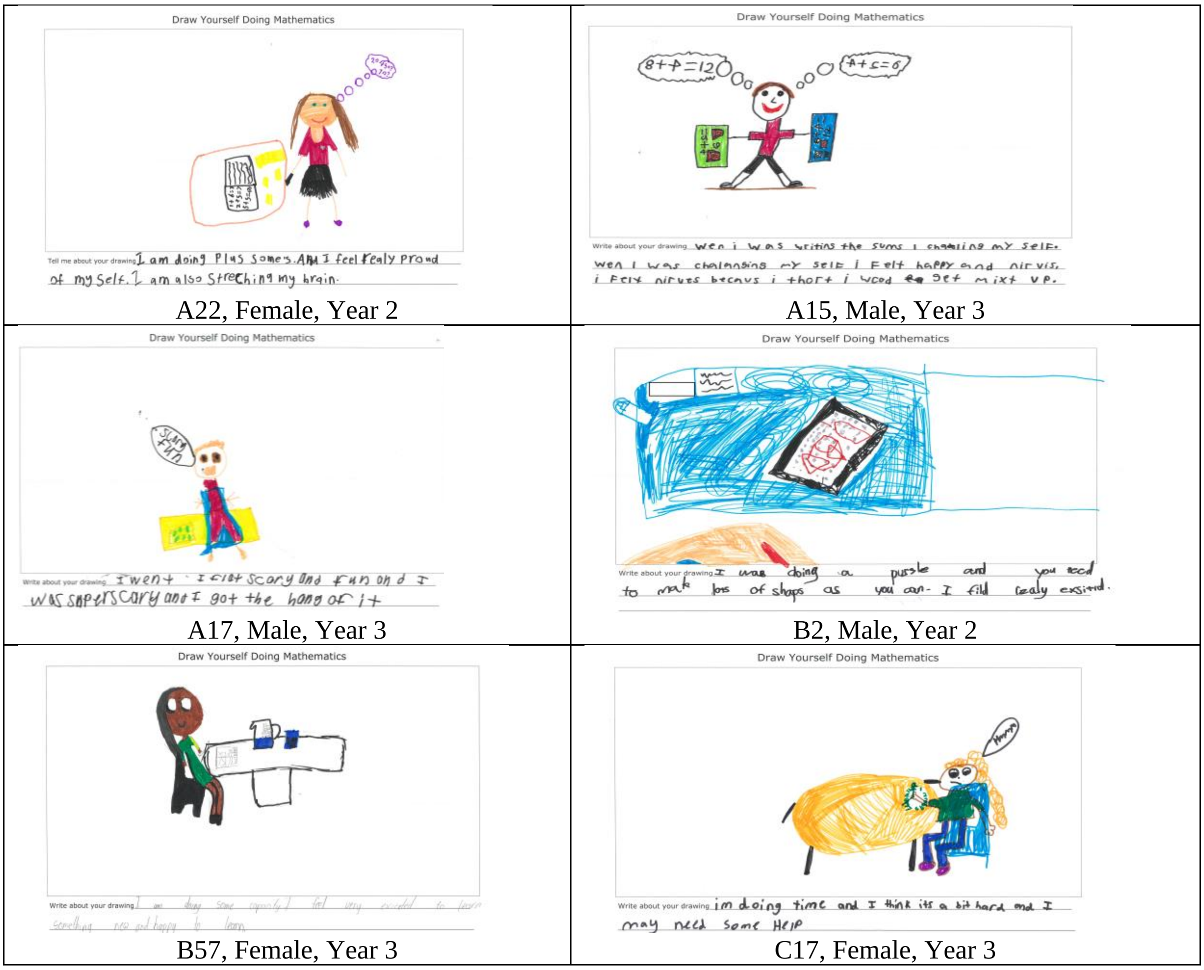


*Figure 1.* Children drawing themselves doing mathematics.

In reporting these results, it is easy to resort to the traditional dichotomy definition of attitudes. However, this overlooks the complexity in describing YCATM and how young children view manipulatives, their emotional response towards their use and how combined with their perceived competence. The findings are reported and discussed using MTMA and Bruner's experiential learning stages to describe children's attitudes.

### *Attitudes Towards Mathematics and Manipulative Use: Enactive Stage*

In the non-lesson context, children connected the use of manipulatives to a variety of mathematical topics, predominately number (operations and place value), followed by 2D shapes, time, reading analogue clocks, and money. Children's Emotional Tendency towards using manipulatives varied greatly depending on the topic and manipulative used, which contributed to children's overall Emotional Sentiment towards mathematics. The majority of children, even children who were classified as having a Negative Attitude towards mathematics, appeared to exhibit Positive Attitudes towards manipulatives during enactive learning experiences. For example, when using manipulatives, A10 showed perseverance and interest in what she was doing, and this was a stark contrast to the other observed learning experiences when A10 exuded disdain and negativity.

In contrast, not all children appeared to appreciate using manipulatives in the same way as A10, leading to negative emotions and views of manipulatives. For example, C9 found the enactive phase frustrating and prohibitive, and this was seen in both the non-lesson and lesson context. During the non-lesson context, C9 did not draw any iconic representations of manipulatives. However, he did speak extensively about using manipulatives. C9 described his 'hate-love-hate' relationship with mathematics, attributing number concepts, particularly multiplication, as the cause of his disdain. C9 provided several examples in a non-lesson context of what he thought was annoying "cause [sic] you need to make like one hundred groups of sixty-five," referring to using counters to represent multiplication as an array. In a lesson context, C9 used manipulatives as an opportunity to disengage by wandering the room, collecting a single counter from several locations before returning to his desk. Another child, C17 (see Figure 1), revealed in the non-lesson context that she "sometimes feel a little bit anxious, anxious where I just want to give up" and related these feelings to using analogue clocks to tell the time. C17 has drawn herself using an analogue clock, stating, "I feel like I need some help, and I need to get my brain thinking more."

### *Attitudes Towards Mathematics and Manipulative Representation: Iconic Stage*

Children's drawings were noteworthy sources to examine the confluence of manipulative use, representation, and attitude formation. The use of iconic imagery in the non-lesson context was documented by children in their written descriptions of their drawings. Figure 1 shows a range of iconic representations of manipulatives representing topics from addition and skip counting, subitising, 2D shapes, capacity and clocks. A range of emotions was depicted in the drawings, ranging from happiness and enjoyment (A22, B2, and B57) to feeling nervous and scared (A15, A17, and C17). These sources provided opportunities for further inquiry. For example, A22 liked to use manipulatives, such as the blocks that she has drawn, to help her find a solution, "well, why I chose to do plus sums is because I really like umm solving them with different things and I especially like using the blocks." A22 was able to represent the manipulatives iconically, seeing value in representing her mathematical thinking in multiple ways, contributing to her Vision of Mathematics and her Perceived Competence.

However, during the lesson context, many children appeared to struggle to create iconic representations of the representations used in the enactive stage. Further, they were yet to understand the notion of a productive struggle. For example, B8, when using Polydrons to model a cube, quickly realised that it was not possible to draw multiple 1:1 scale iconic representations of the physical representations that he was creating on an A4 piece of paper. This impediment led to outward signs of frustration, anger and ultimately not pursuing more than two possible solutions. Similarly, B52 struggled to create iconic representations of familiar objects during a fraction learning experience. While other children in B52's group acted as enablers and shared their strategies on how they substituted the familiar object with a

diagram, B52 was adamant that it was not achievable. It appears that transitioning from enactive to iconic caused some children to outwardly exhibit negative emotions resulting in disengaging, especially in tasks with higher levels of cognitive demand.

The transition between enactive and iconic experiences was exacerbated by introducing iconic experiences before children were ready or had developed the necessary conceptual understanding, as seen in the following vignette. C9, C11 and C21 could identify 3D solids when engaging in a learning experience with 3D wooden attribute blocks and use these blocks to identify and describe related objects in their environment. The children used the attribute blocks to determine the number of faces, corners, or edges by rotating the block to assist in identifying features. However, all struggled with visualising iconic representations of 3D solids as a 2D representation in the form of a net. This struggle is contextualised in the discussion.

### *Attitudes Towards Mathematics and Manipulative Representation: Symbolic Stage*

Children created symbolic representations of manipulative use in both the lesson and non-lesson context. As seen in Figure 1, children wrote number sentences to accompany the representation on manipulatives. For example, A15 writing the number sentence 4 + 2 = 6 to match the numbers shown on the dice that formed part of the enactive stage. In doing so, we see the number formation, including number reversal and how he feels about the symbolic stage, where he admits to feeling nervous about getting “mixed up.”

In a non-lesson context, children with Positive Attitudes depicted more complex number sentences and were able to describe mathematical processes to perform the operations depicted. Children with Positive Attitudes were more likely to draw iconic representations of manipulatives and showed how they enacted their use in their drawings. For example, B1 (Year 2) wrote 636 + 636 and depicted the process of partitioning (“chunking”) to work out the answer. Other children used number lines to show repeated addition of two-digit numbers, while others drew MAB (longs and units) to show the processes they used to add numbers. These children were confident in using multiple representations (an indicator of the TTP) to answer the questions (an indicator of their SC). Further, children with a Positive or Extremely Positive Attitude classification embraced the notion of creating and developing their own symbolic representations, confidently showing their mathematical thinking and working. Children with Neutral, Negative and Extremely Negative Attitude classifications are yet to develop this confidence. Children with a Negative and Neutral Attitude classification struggled in transitioning from physical representations (manipulatives) to visual and symbolic representations.

The lesson context provided further examples of how children used symbolic representations of manipulatives and how creating symbolic representations influenced their attitude towards mathematics. The following vignette is from two children in the same class attempting the same task that required multiple ways of adding numbers to 12. A22 worked independently on the task, regularly making statements about what she was doing. Her self-talk was audible to others but appeared to be directed at no one in particular. The child worked on the task, continuing to self-talk when the teacher clarified the instruction about the task, stating that a number c only be used once. A22 stood up to get an eraser from a different desk and returned to her seat, uttering a mild expletive before erasing some of her work. A22 resumed the task independently and soon resumes the self-talk uttering “9 + 2 + …” and “I’ve got two done,” quickly followed by “I’ve got four questions done.” The child continued to work on the task. In contrast, A18 was reluctant to make a start and appears to be avoiding and delaying work. A18 pushed her chair backwards, away from the table, physically distancing herself from her work, finding other unrelated reasons for not completing the task. Even with teacher prompting, A18 was hesitant and exhibited signs of discomfort and distress. She stood in the doorway, arms folded on her chest, frowning and huffing. She moved further into the

corridor so that she could not be seen, occasionally glancing back into the room, remaining there for approximately two minutes. A18 returned to her desk, stating that she had only got one number sentence (3 + 4 + 5) with two children suggesting two solutions. A18 ignored their assistance and began counting out some pop sticks and proceeded to write a second number sentence in her workbook, opting not to share her solutions with other children.

## Discussion

The use of manipulatives was not done in isolation; as previously discussed, enactive manipulative use accompanied by symbolic and iconic representations. Attitudes towards mathematics varied in all three stages of Bruner's experiential stages of learning. The largest variation in attitudes were noted in transitioning between the iconic and symbolic stages with several factors identified that contributed to this variation.

The transition from physical to internal representations (Goldin & Shteingold, 2001) via the enactive, iconic and symbolic stages (Bruner, 1996) influenced children's attitudes. The transition between representations is a vital development in the learning and acquisition of mathematics, especially as one of the goals of mathematics education is for "children to create and think critically about mathematics" (Perry & Atkins, 2002, p. 201). Children need time to develop confidence in using physical representations before introducing "conventional notation" (Perry & Atkins, 2002, p. 201). Further, children need the connection between the enactive, iconic and symbolic representations or informal and formal representations to be made explicit (McNeil & Jarvin, 2007). Conversely, spending too long on a particular representation or method can result in frustration and boredom in children, causing Negative or Neutral Attitudes towards mathematics.

During the enactive phase, manipulatives made the intended learning accessible and enjoyable, thereby fostering Positive Attitudes towards mathematics. Moch (2002) found similar results, reporting that children who were previously reluctant were more eager and enthusiastic.). The eagerness of the children in this study manifested in many ways, where we see children such as A10 and A22 wanting to complete tasks that involve the use of manipulatives where previously, there were reluctant to engage. In contrast, some children in the study appeared reluctant to use manipulatives to work through cognitively demanding tasks, even after the teacher prompted the use of specific manipulatives. It seemed for these children, that the use of manipulatives was viewed as a last resort and not a useful mathematical tool. For a minority of children who described discomfort with mathematics, manipulatives were a tool that was used to actively or passively disengage from mathematical learning experiences. McNeil and Jarvin (2007) found similar results, reporting that manipulatives, in some cases, can be detrimental to learning. Further research is recommended to identify the relationship between disengagement and manipulatives, as it is possible that students are unfamiliar with the manipulative and do not know how to use it to support their conceptual development.

The transition between enactive and iconic was further inhibited by introducing the iconic representation of 3D solids before it was formally introduced, which is currently located in the Year 5 *Australian Curriculum: Mathematics*. Consequently, introducing iconic representations too early was shown to cause confusion and frustration. The children's Emotional Tendency was not the only dimension of attitude impacted. Children's Mathematical Mindset and Self-concept were also negatively impacted.

## Conclusion

The Modified Three-dimensional of Attitude was a flexible framework that moved beyond the identification and classification of children's attitudes to analyse factors that influenced attitudes. The children in this study used many types of representations, with manipulatives

emerging as a predominant representation. Investigating how students use and create with manipulatives went beyond gaining insights into their mathematical thinking. Rather, insights were gained about how children viewed manipulatives, how children felt about and their confidence in using these materials. Several themes emerged when investigating the confluence of attitudes towards mathematics and the use of manipulatives. When used effectively and timely, manipulatives provide a solid basis for developing flexible external and internal representations of mathematics and contribute to the formation of Positive Attitudes towards mathematics. Other variables, however, influenced how children viewed and used manipulatives. Disengagement was documented in several cases, where children used manipulatives as a façade for doing mathematics and were a means to actively and passively disengage. A third theme related to the transition between enactive, iconic and symbolic representations was noted and how this can contribute to Negative Attitudes towards mathematics. These contextual variables and the cues and signs children provided need to be considered when planning mathematical learning experiences.

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